Dynamic Analysis of Concrete Pavement under Moving Loads

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Abstract

Based on a Finite Element formulation of thick plate on elastic foundation, parametric studies are performed to investigate some significant aspects of the dynamic behavior of rigid concrete pavements. Since the 2-parameter soil provides a rational and computationally efficient model of the sub-soil, it is used in this work as the sub-grade soil model and the differences from the response of the well-known Winkler foundation is investigated. The parametric studies also include the difference between static and dynamic response and investigate the effects of structural damping, vehicular speed and soil stiffness on the response of concrete pavements due to single wheel as well as HS20 loading. The wheels are moved over the edge and centerline of the pavement and the resulting deflections and principal tensile stresses are calculated from a dynamic time domain analysis. Results from the numerical analyses indicate that the Winkler soil model may overestimate the structural responses and significantly over design the pavement as a result. The effects of soil stiffness and structural damping are found to be significant, as is the importance of dynamic analysis. However, the effect of vehicular speed is found to be relatively less significant in this study.

Keywords: Concrete pavement; Dynamic analysis; HS20; Winkler foundation; Two-parameter soil

Introduction

Pavements are an essential feature of the urban communication system and provide an efficient means of transportation of goods and services. Depending on its rigidity compared to the subsoil, pavements are classified as flexible, rigid and semi-flexible.

The conventional methods of rigid pavement design are based on the closed-form solutions obtained from the static analysis of infinitely long plates resting on an elastic foundation (Westergaard 1926). The actual discontinuous nature of pavement systems is disregarded in this approach. Further, the dynamic effects of moving vehicles are accounted for indirectly by applying an impact factor.

The dynamic response of beams and plates resting on an elastic foundation subjected to moving loads was also studied by several researches in the past. However, most of these studies used the well-known model of elastic foundation developed by Winkler, which assumes the foundation to behave like independent discrete springs whose stiffness is known as the 'Modulus of sub-grade reaction', usually denoted by '*k*'.

However, the Winkler model neglects the interconnection among the soil layers and as a result may impose some serious limitations in the physical modeling of the sub-soil system. These limitations can be improved by modeling the sub-grade as a two-parameter medium, which provides shear interaction between individual spring elements. In this work the sub-soil, assumed to be a uniform deposit over a very stiff half-space, is approximately modeled as a two-parameter medium whose properties are based on the work by Vlasov & Leontev (1966).

This study deals with the dynamic analysis of rigid pavements under the action of moving vehicular loads. The pavements are modeled as thick plates discretized by finite elements and the action of vehicular loads is modeled as a series of moving wheel loads similar to the HS20 loading suggested by AASHTO.

Received 28 February 2004, Accepted 10 August 2004

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Modeling of the Sub-Soil and Foundation

The Winkler Foundation Model

For the analysis of beams and slabs resting on a soil medium, engineers have been using the classical mathematical model called Winkler model (Winkler 1867), where the behavior of the soil is simplified by means of fictitious independent closely spaced springs placed continuously underneath the structure. The corresponding spring constant *k* is called the 'Modulus of sub-grade reaction' of the soil.

Recommendations for the values of *k* are found in the works by Biot (1937), Terzaghi (1955), Vesic (1973) and others. Significant applications of the Winkler-type soil model have been shown in more recent works by Bay et al. (1996), Kim & Roësset (1998), Huang & Thambiratnam (2002) and others. A comprehensive text by Hetenyi (1946) on 'Beams on Winkler Foundation' is used internationally.

So far, based on this concept, many computer codes have been developed for the analysis of beams and slabs on an elastic foundation; the user of the code has to determine a suitable value of *k* to represent the soil. There is no easy way to determine this value of *k* because its value is not unique for a given type of soil, as suggested in some textbooks on foundation engineering. Usually, the soil is stratified, having different thickness, and the value of an equivalent *k* has to be at least a function of the thickness of the soil layer, even when its materiel properties remain the same. The larger the thickness, the lower is the value of *k*.

Many researchers while calculating values of *k* (e.g., Biot 1937, Terzaghi 1955, Vlasov & Leontev 1966, Vesic 1973) have proved its lack of uniqueness and have suggested that its value has to be augmented on edges of the slab, emphasizing the need for more research on this topic. In other words, the value of *k* varies in the domain of the slab for different material and geometric properties of the soil.

In addition to the lack of uniqueness of *k*, the Winkler model has physical limitations. Since it models the soil as individual springs without any interconnection, an obvious deficiency is that the un-stressed soil (beyond the range of the loaded area) is also free of any deformation. This often violates the compatibility conditions as well and may lead to serious errors.

Two-Parameter Foundation Model

A number of different two-parameter models have been proposed in the past, including works by Pasternak (1954), Vlasov & Leontev (1966). Several researchers have demonstrated have demonstrated the capability of two-parameter models in representing the soil medium for static (e.g., Yang 1972) as well as dynamic (Zaman et al. 1991, 1993) problems. A method for calculating the parameters of the model was presented by Vallabhan et al. (1991).

A single-layer elastic foundation of finite thickness H is considered. The subject matter will be restricted to the problems where the horizontal displacement is negligible. With this assumption, the distribution of displacements and the normal stress in the vertical z direction over the height H is determined by a function $\psi(z)$. In addition, it is assumed that the shear stress at the interface between the compressible layer and the rigid base equals to zero. The vertical displacement could therefore be expressed as,

$$w(x,y,z) = W(x,y)\ \psi(z) \quad (1)$$

in which $W(x,y)$ = vertical deflection of the foundation at surface level and $\psi(z)$ = function for transverse distribution of the displacements over the depth of soil stratum, chosen in accordance with the nature of the foundation.

For a relatively thin compressible layer of foundation, the variation of the normal stress with depth may be small and therefore could be considered as constant with depth. Under these conditions, the form of the $\psi(z)$ function could be approximated by a linear function.

However in a thick layer of foundation, the normal stresses vary considerably with depth and therefore the form of $\psi(z)$ function must take a different form. In order to account for the decrease of the displacement and the normal stress with depth, the $\psi(z)$ function could be selected as

$$\psi(z) = \sinh\{\gamma(H-z)/m\}/\ \sinh(\gamma H)/m) \quad (2)$$

where m is the dimension of the subsequently considered plate and γ is a constant determining the rate of decrease of the displacements with depth as with this form of $\psi(z)$, it is seen that the normal stresses vary with depth as the hyperbolic cosine. This form of $\psi(z)$ can also be used for the semi-infinite layer where H becomes infinity.

Depending on the nature of the particular problem, many analytical expressions in addition to Eqs. (1) and (2) can be selected. In fact, the expression could be based on experimental data of normal stress distributions. Based on the conventional constitutive and compatibility relationships, together with the displacement of all points expressed by Eq. (1), the condition of equilibrium of the foundation model to an externally distributed load $q(x,y)$ on the surface can be expressed as

$$2t\ \Delta^2 w(x,y) + k\ w(x,y) = q(x,y)\ \psi(0) \quad (3)$$

in which, $\Delta^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplace operator, and

$$k = E_0/(1-\nu_0^2) \int \psi(z)^2\, dz$$

$$t = E_0/4(1+\nu_0) \int \psi(z)^2\, dz \quad (4)$$

are the two elastic of the single- layer foundation; in which $\int$ is an integration over the entire depth of the soil layer; i.e., for values of z between 0 and H

$E_0 = E_s/(1-\nu_s^2)$ and $\nu_0 = \nu_s/(1-\nu_s)$

where E_s and ν_s are respectively the modulus of elasticity and Poisson's ratio of the foundation.

For the transverse displacement function described by Eq. (2), the two parameters k and t become

$$k = \{E_0\gamma/2m(1+\nu_0^2)\}\{\sinh(\gamma H/m)\cosh(\gamma H/m) + \gamma H)/m\}/\sinh^2(\gamma H/m)$$

$$t = \{E_0\, m/8\gamma(1+\nu_0)\}\{\sinh(\gamma H/m)\cosh(\gamma H/m) - \gamma H)/m\}/\sinh^2(\gamma H/m) \quad (5)$$

However, the present work does not model the sub-soil as a three-dimensional elasto-plastic or even an elastic medium, because such models are considered too expensive numerically. The two-parameter model is considered to be a rational compromise between the too simplistic Winkler model and the computationally inefficient three-dimensional model.

Static and Dynamic Analysis of Plate on Two-parameter Foundation

For a rectangular plate resting on a single-layer two-parameter elastic foundation and traversed by a moving load, the governing differential equation is expressed as

$$D\ \Delta^4 w(x,y) - 2t\ \Delta^2 w(x,y) + k\ w(x,y) = q(x,y) \quad (6)$$

where D is the plate rigidity and $q(x, y)$ is the dynamic sub grade-pavement-vehicle interaction force transmitted to the plate.

The governing differential equation has to be solved by applying appropriate boundary conditions. For a rectangular plate element resting on a two-parameter foundation, Eq. (6) can be transformed into the following stiffness matrix formed by adopting the variational principle:

$$\{[k_0] - [k_1] + [k_2]\}\{d\} = \{Q\} \quad (7)$$

where $[k_0]$ is the element plate stiffness matrix, $[k_1]$ and $[k_2]$ are the foundation stiffness matrices corresponding to the foundation parameters t and k respectively, $\{Q\}$ is the element nodal force vector and $\{d\}$ is the element nodal displacement vector. The stiffness matrices and the force vector defined in Eq. (7) can be derived in terms of foundation properties. The detailed expressions are available in literature (e.g., Zaman et al. 1993) and not repeated here.

The matrix formed by the combination of $[k_0]$, $[k_1]$ and $[k_2]$ is the stiffness matrix $\boldsymbol{K}$ of the plate-foundation system. In addition, the mass matrix $\boldsymbol{M}$ and damping matrix $\boldsymbol{C}$ are needed for the dynamic analysis of the system. It may be mentioned here that all the matrices have been computed by Gaussian integration and the consistent mass matrix has been used for $\boldsymbol{M}$. Once they are formed, the dynamic analysis in the time domain can be carried out numerically. The most widely used numerical approach for solving dynamic problems is the Constant Average Acceleration method (i.e., Newmark-β method with $\alpha = 0.50$ and $\beta = 0.25$), which has

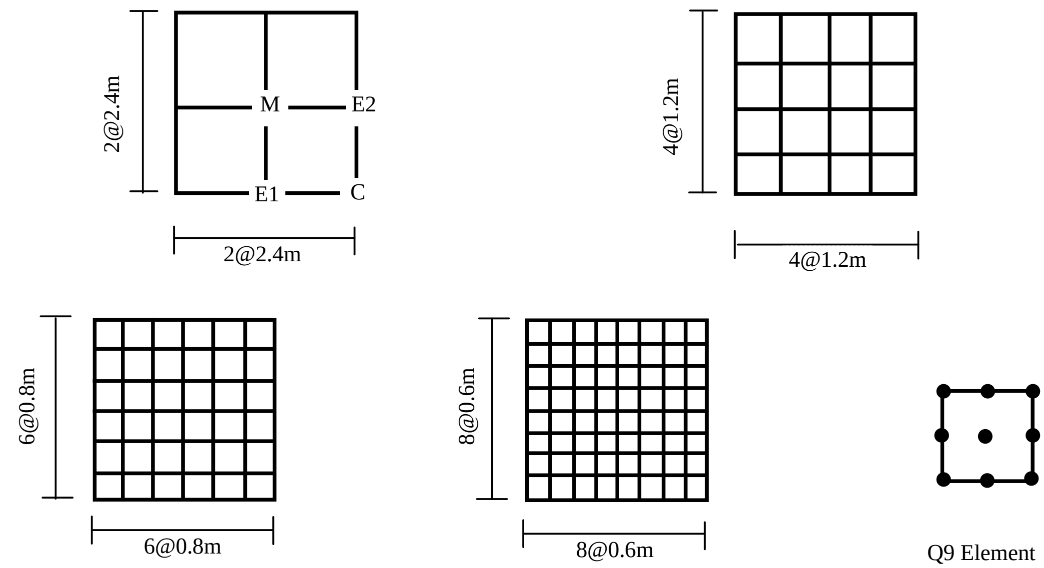

Fig. 1 Finite Element models of the pavement

been used here also with a time step of 0.001 seconds.

Material Properties and Structural Models

The important material properties used in this work include the modulus of elasticity (e.g., 20000 Mpa for concrete, 50 MPa for soil), Poisson's ratio (0.25 for concrete, 0.30 for soil), and modeling parameter for the 2-parameter soil (γ = 1.20). The structural properties include the size (4.8 m × 4.8 m) and thickness (23 cm) of the concrete pavement used here and the depth of the uniform soil layer beneath the foundation (30 m).

The Finite Element models used for the pavement are shown in Fig. 1. The mesh arrangements used for the convergence study of the results are (2×2), (4×4), (6×6) and (8×8), the (4×4) model being used for the rest of the studies as well. The quadratic Q9 element used for the Finite Element modeling is also shown in Fig. 1.

The critical points used for the deflections and stresses are also shown in the figure. The point C refers to the 'corner point' and E1 an 'edge point'. Both of them are used to study the deflections and stresses for edge loading. The point E2 also refers to an 'edge point' and M the 'mid point' of the pavement. They are used for the calculation of deflections and stresses for centerline loading.

For the dynamic analyses, the standard AASHTO loading HS20 is passed over the edge as well as the centerline of the pavement. It may be mentioned that the HS20 loading consists of two three-wheel systems 2 m apart along the width of the vehicle. Each system consists of a 20 kN load followed by two 80 kN loads 4.2 m apart along the length of the vehicle.

Convergence Study for the Finite Element Models

Before detailed parametric studies for the plate elements, the accuracy of the Finite Element model is studied. In order to do that, various Finite Element models are used with increased mesh density. The three models shown in Fig. 1 are used for the convergence study of the critical deflections and stresses. For this purpose, a single 20 kN wheel is passed along the edge of the pavement.

Both the static and dynamic analyses are performed for the Winkler soil model while only the dynamic analysis is performed using the 2-parameter soil model.

Results from the numerical analyses (for the 'edge point' E1 and 'corner point' C) are summarized in Table 1, which shows the excellent convergence of the Finite Element models with increased mesh density. In fact the deflections for the denser meshes [i.e., (6×6) and (8×8)] are very similar, although most results for the less dense (2×2) mesh are somewhat different, particularly for the dynamic analyses using 2-parameter soil model.

The convergence of the Finite Element models can be studied by observing the variation of results with the mesh size. For example, Fig. 2 shows the variation of corner deflection and edge deflection obtained by dynamic analysis as the 4 kips wheel passes over the edge of the pavement while it is supported on 2-parameter soil.

The 'relative mesh size' is obtained by the inverse of the number of elements in the x and y-direction. Thus the 'converged' results are obtained when the 'relative mesh size' is zero; i.e., the number of elements is infinite. The plots show that the variations are almost linear, from the 'converged' results are obtained by two methods (Conv1 and Conv2), as shown in Table 1. Conv1 refers to the 'converged' results by fitting the curves in Fig. 2 to best-fit straight lines, while Conv2 represents the values by simple linear extrapolation of the results from the (2×2) and (8×8) meshes. The results from these two methods show very good agreement, which validates the use of the relatively simpler second method (Conv2) for the subsequent studies. Therefore, for all the subsequent studies (i.e., time series plots or their maximum values), the 'converged' deflections as well as stresses are referred to the values obtained by linear extrapolation of these two sets of results.

Results from Numerical Analyses

As mentioned, the 'converged' results obtained by the Conv2 approach shown before (linear extrapolation) is used for the subsequent numerical studies because the results from this arrangement are considered to be sufficiently accurate.

Table 1 Maximum deflections for different mesh arrangements
(Wheel Load = 20 kN, V = 100 kph, Damping Ratio = 5%)

Numerical Model	Item	2×2	4×4	6×6	8×8	Conv1	Conv2
Winkler Soil Static Analysis	δ_C	1.319	1.362	1.385	1.398	1.419	1.424
	δ_{E1}	0.424	0.479	0.499	0.507	0.535	0.535
Winkler Soil Dynamic Analysis	δ_C	1.485	1.561	1.561	1.565	1.601	1.592
	δ_{E1}	0.507	0.560	0.575	0.581	0.608	0.606
2-Parameter Soil Dynamic Analysis	δ_C	0.222	0.284	0.298	0.303	0.334	0.330
	δ_{E1}	0.150	0.178	0.187	0.195	0.208	0.210

[All deflections are in mm]

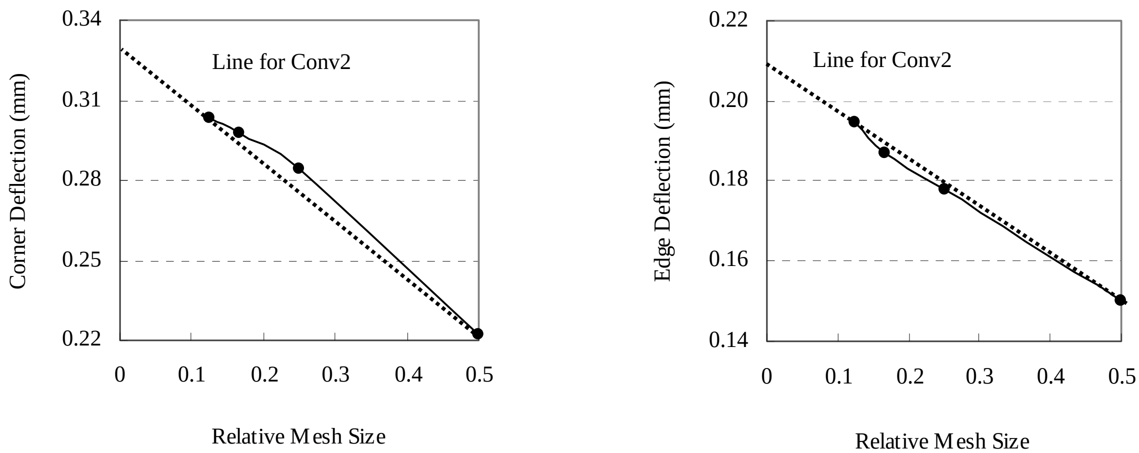

Fig. 2 Convergence study of deflections (2-parameter soil)

To observe the characteristics of the fundamental response functions, a single 20 kN wheel is passed through the edge and centerline of the pavement. Once again, both the static and dynamic analyses are performed for the Winkler soil model while only the dynamic analysis is performed using the 2-parameter soil model. The maximum values of deflection and maximum tensile stress at critical points are summarized in Table 2.

The significant observations from this preliminary analysis include the higher stresses at the pavement edge compared to mid-span and at corner compared to the edge, the higher stresses for Winkler model compared to 2-parameter soil and the difference between static and dynamic analyses. Even for this single wheel loading, the results using Winkler soil model shows that the principal tensile stress in the corner of the pavement reaches about 1 Mpa for edge loading. This stress is of the same order as the tensile strength of concrete, which suggests a possible tensile failure of concrete. However, the stresses calculated by the 2-parameter model are much lower. This indicates very significant over-design of pavements using the Winkler soil model.

The parametric studies for the dynamic analyses for HS20 loading are different here, with Case1 referring to the dynamic analysis of pavements with 5% damping resting on 2-parameter soil with elasticity modulus of 50 MPa and wheels (HS20 loading) moving at a speed of 100 kph. Case2 to Case6 consist of some variations of the parameters mentioned in Case1. Case2 refers to the results for an undamped structure with all other properties remaining the same, Case3 shows results from static analyses, the foundation is changed to Winkler model in Case4, the vehicular velocity is reduced from 100 kph to 50 kph in Case5, while the elastic sub-grade modulus is halved in Case6.

Only the results for the corner deflections and stresses are shown for the edge loading and midspan deflections and stresses for the centerline loading. The maximum values of deflection and stress at critical points are summarized in Table 3 while Fig. 3 to Fig. 8 show the time series variations of the corner deflections for the wheels passing along the edge of the pavement. The results from the analyses show the significance of structural damping and dynamic analysis, soil model (Winkler vs. 2-parameter, particularly for edge loading) and stiffness of the sub-grade soil. However, the effect of decreasing the vehicular speed from 100 kph to 50 kph is found to be insignificant here.

Conclusions

This paper presents some significant results from a detailed study (Rahman 2003) on the dynamic analysis of rigid pavements. The soil models used in this work are the well-known Winkler model and an improved 2-parameter model incorporating the interaction between soil layers. The main conclusions of this study are

1. For a wheel load passing along the edge of the pavement, the Winkler model predicts substantially higher deflections and stresses at the corner (C) than at the middle of the edge (E1). However, these are not so significant for the 2-parameter model, mainly due to its additional corner stiffness and interconnection with surrounding soil. For wheel loads passing through the pavement centerline, a similar comparison can be made between the results at edge (E2) and middle (M).

Table 2 Maximum deflections and stresses for a single wheel
(Wheel Load = 20 kN, V = 100 kph, Damping Ratio = 5%)

Numerical Model	**Wheel along Edge**				**Wheel along Centerline**			
	δ_C	δ_{E1}	σ_C	σ_{E1}	δ_{E2}	δ_M	σ_{E2}	σ_M
Winkler Soil Static Analysis	1.424	0.535	0.919	0.385	0.575	0.154	0.122	0.145
Winkler Soil Dynamic Analysis	1.592	0.606	0.989	0.614	0.622	0.192	0.193	0.230
2-Parameter Soil Dynamic Analysis	0.330	0.210	0.218	0.387	0.225	0.119	0.143	0.146

[All deflections are in mm and stresses are in MPa]

Table 3 Maximum deflections and stresses for HS20 loading

Numerical Model	**Wheels along Edge**		**Wheels along Centerline**	
	δ_C	σ_C	δ_M	σ_M
Case 1	1.328	0.486	0.466	0.715
Case 2 (Undamped)	1.579	2.117	0.558	1.256
Case 3 (Static Analysis)	1.288	0.064	0.436	0.281
Case 4 (Winkler Soil)	6.522	2.396	0.829	1.362
Case 5 (V = 50 kph)	1.411	0.483	0.477	0.705
Case 6 (E_s = 25 MPa)	2.319	0.976	0.934	0.900

[All deflections are in mm and stresses are in MPa]

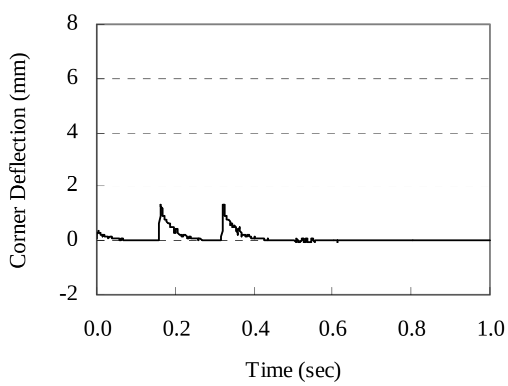

Fig. 3 Corner Deflection for Case 1
(Model Conditions)

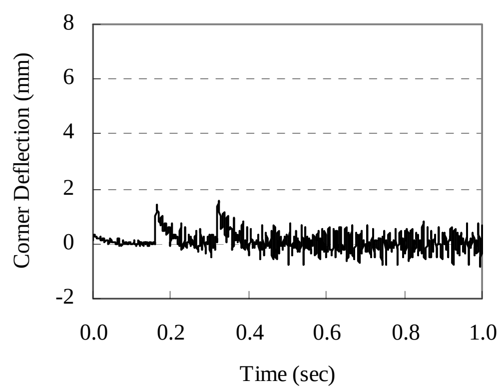

Fig. 4 Corner Deflection for Case 2
(Undamped Dynamic)

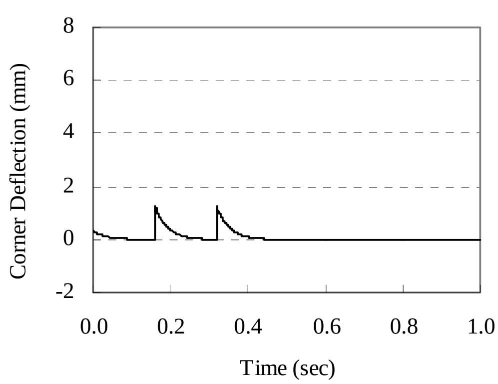

Fig. 5 Corner Deflection for Case 3
(Undamped Static)

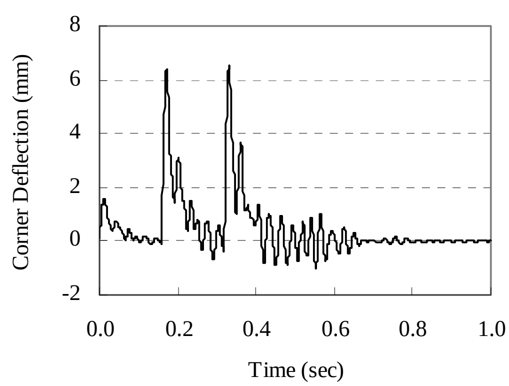

Fig. 6 Corner Deflection for Case 4
(Winkler Foundation)

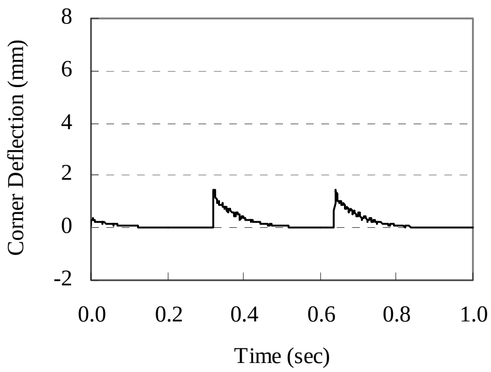

Fig. 7 Corner Deflection for Case 5
(V=50 kph)

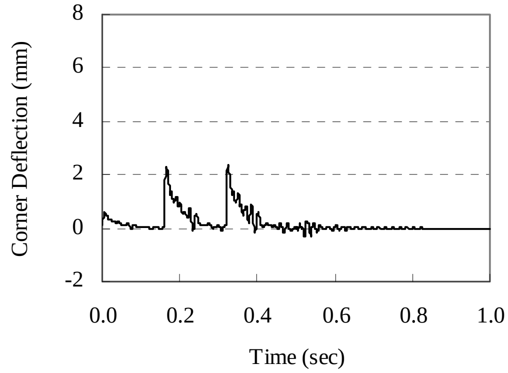

Fig. 8 Corner Deflection for Case 6
(Softer Soil)

2. The deflections and stresses from the Winkler model are significantly higher than the 2-parameter results, both for the edge loading as well as the centerline loading. This indicates a possible over-design of pavements using the Winkler soil model. In this study, the maximum tensile stress for the pavement with (5% damping) over Winkler soil went upto about 1 MPa for 20 kN wheel, but the corresponding maximum stress for the pavement over 2-paramter soil was less than 0.4 Mpa.
3. The effect of damping is found significant, with results from the undamped structure being much higher than the damped structural responses due to the large free vibration for undamped system. Although real structures are almost never undamped, this result demonstrates the significance of damping ratio on the structural response, particularly if the ratio is small.
4. The results from static analyses are much lower compared to dynamic results particularly for the HS20 loading, which shows the importance of dynamic analysis in the design of pavements.
5. The effect of changing the vehicular speed is found to be insignificant here. In this study, the results do not change much even after reducing the vehicular speed from 100 kph to 50 kph. In fact, the maximum deflections or stresses appear to be slightly smaller in some cases and slightly greater in others.
6. The deflections and stresses depend significantly on the stiffness of the sub-grade. In this study, they are almost doubled by using a soil whose elasticity modulus is half of the original model.

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