NONLINEAR PROPERTIES OF REINFORCED CONCRETE STRUCTURES

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Abstract: *The importance of various nonliearities involved in the static and dynamic analyses of Reinforced Concrete structures is investigated in this paper. The nonlinearities studied here are geometric (caused by large deformations and consequent effect on the elastic properties of the structure) as well as material (due to the nonlinear stress-strain relationship of concrete and steel). In the first part of the paper, the nonlinear moment-curvature relationship of arbitrary Reinforced Concrete cross-sections is developed numerically using nonlinear stress-strain relationships for concrete and steel. The relative importance of geometric and material nonlinearity is studied for a simple 2-storied frame under static vertical load. Although the effect of material nonlinearity is more important in most of the cases studied here, the geometric nonlinearity becomes significant at higher loads. The effect of axial load on the moment-curvature relationship is studied, and the effect of typical axial loads on the flexural behavior of column is found to be significant. The shear strength of the beams and columns (obtained from empirical equations suggested in the ACI Code) prove to be very important here. Using the nonlinear sectional properties thus obtained, the nonlinear structural dynamic analyses of the building are performed subjecting the structure to seismic vibrations using nonlinear structural dynamics. Recorded ground motion data from two major earthquakes of the past; e.g., the El Centro earthquake in USA (1940) and the Kobe earthquake in Japan (1995) are used in the dynamic analyses. The results show the difference between the linear and the nonlinear structural response.*

Introduction

Reinforced Concrete (commonly known as RCC for Reinforced Cement Concrete) is a widely used construction material in many parts the world. Due to the ready availability of its constituent materials, the strength and economy it provides and the flexibility of its forms, RCC is often preferred to steel, masonry or timber in building structures.

From a structural analysis and design point of view, RCC is a very complex composite material. It provides a unique coupling of two materials (concrete and steel) with entirely different mechanical properties. They combine to produce a composite, which behaves like an elasto-plastic material that responds differently to tensile and compressive stresses. Also, due to the cracking of concrete, even the sectional and therefore the structural properties depend on the nature and magnitude of the applied loads.

All these complexities can manifest particularly when the structure is subjected to dynamic (time-varying) loads like wind, storm, wave and strong ground motions like earthquake. Despite the importance of the material nonlinearities or the time-varying properties, they are rarely considered in the analysis and design of structures made of RCC.

The unsatisfactory behavior of several RCC buildings under severe loads like storms and earthquakes call for more rigorous research work on the elasto-plastic behavior of RCC under dynamic loads. The considerable amount of experimental work notwithstanding, there is a growing need for theoretical and computational work to form a more rational model of RCC. The existing RCC design codes do not approach these possible causes of structural failure with enough importance. This study aims to contribute to the existing knowledge of the elasto-plastic dynamic behavior of RCC.

Based on a rigorous nonlinear structural model of the material and also considering the geometric nonlinearities, parametric studies are performed in this study to investigate some important details of the behavior of RCC. A simple 2-storied, 2-dimensional frame is taken for nonlinear static structural analysis under increasing vertical load and nonlinear dynamic analysis under combined vertical load and seismic vibrations. Various aspects of the nonlinear response of RCC are studied. These include the relative importance of geometric and material nonlinearity, the effect of axial force on the Moment-Curvature (M-ϕ) relationship, importance of flexural strength and shear strength and the difference between linear and nonlinear dynamic analysis.

Nonlinearities in RCC

Although the literature on concrete (Neville, 1963) and RCC structures (Winter & Nilson 1983, Pillai & Menon 1998, Park & Paulay 1975) is full of elastic parameters like proportional limit and modulus of elasticity, the behavior of RCC cannot be modeled properly by linear elastic behavior. Recognizing this, the design of RCC structures has gradually shifted over the years from the 'elastic' Working Stress Design (WSD) to the more rational Ultimate Strength Design (USD). The nonlinearities in RCC members can be geometric as well as material. Both of these become more important at higher deformations.

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Geometric Nonlinearity

Linear structural analysis is based on the assumption of small deformations and linear elastic behavior of materials. The analysis is performed on the initial undeformed shape of the structure. As the applied loads increase, this assumption is no longer accurate, because the deformations may cause significant changes in the structural shape. Geometric nonlinearity is the change in the elastic load-deformation characteristics of the structure caused by the change in the structural shape due to large deformations. While this requires complicated formulation, reasonable accuracy can be achieved by suitable approximation of the problem.

For example, in one-dimensional flexural members modeled by the 'Euler-Bernouli beam', the geometric nonlinearity can be reasonably represented by approximating the strains up to second order terms. This causes a change in the Stiffness Matrix (with additional nonlinear terms, i.e., function of the displacements) and the resulting structural analysis needs to be performed by iterative methods, like direct iteration (Picard method) or the Newton-Raphson method. These numerical methods are well known and are available in standard texts on structural analysis (Crisfield 1991, Reddy 1993).

In RCC structures, among the various types of Geometric nonlinearity, the structural instability or Moment magnification caused by large compressive forces, stiffening of structures caused by large tensile forces, change in structural parameters due to applied loads (e.g., leading to changed damping or parametric resonance) are significant.

Material Nonlinearity

Concrete and steel are the two constituents of RCC. Among them, concrete is much stronger in compression than in tension (tensile strength is of the order of one-tenth of compressive strength). While its tensile stress-strain relationship is almost linear, the stress-strain relationship in compression is nonlinear from the beginning.

Several researchers have worked on the nonlinear stress-strain relationship of concrete (Hognestad & co-workers 1952, 1955, 1961, Rusch 1960, Kaar 1978). Among them, the Hognestad model (Fig. 1) has been chosen in this work. It approximates the stress-strain relationship by a parabola up to the ultimate strength (f_c') and a straight line beyond that up to the crushing of concrete. The maximum crushing strain (ε_u) and the strain at ultimate strength of concrete (ε_0) are about 0.003 and 0.002 respectively.

Steel, on the other hand, is linearly elastic up to a certain stress (called the proportional limit) after which it reaches yield point (f_y) where the stress remains almost constant despite changes in strain. Beyond the yield point, the stress increases again with strain (strain hardening) up to the maximum stress (ultimate strength, f_{ult}) when it decreases until failure at about a stress (f_{brk}) quite close to the yield strength. The elastic-perfectly-plastic (EPP) model for steel (Fig. 2), which is used in this work, assumes the stress to vary linearly with strain up to yield point and remain constant beyond that.

Since concrete and steel are both strongly nonlinear materials, the material nonlinearity of RCC is a complex combination of both. They are approximately considered in the USD method for RCC design. In this work, they are used to develop the Moment-Curvature relationship for an arbitrary RCC section, beginning with a stage where the section is uncracked, up to failure (a stage when the bending moment of the section decreases with increased curvature).

Moment-Curvature Relation for Arbitrary RCC Section

The M-ϕ relationship of arbitrary RCC sections can be derived numerically by the application of simple principles of Strength of Materials. As shown in Fig. 3, the arbitrary area can be divided into a number of segments. For a given curvature, the position of the neutral axis can be determined by trial and error; i.e., assuming a neutral axis, calculating the strain and stress at various points of the section and equating the compressive and tensile forces.

Once the neutral axis is chosen, the moment M can be calculated easily by summing the moments of all the forces on the section. This method can also be used when there is an extra axial load on the section, as is common for most structural members, particularly building columns.

In this work, this procedure is used to derive the stress-strain relationship for various RCC cross-sections

1. A 16″ deep T-beam section (width 20″ at top and 10″ at bottom and 3″ slab thickness) with two layers of #6 bars (3 at bottom and 4 at top)
2. A 10″×10″ column section with two layers of #7 bars (2 at bottom and 2 at top)

In both cases, the ultimate strength of concrete (f_c') is assumed to be 3 ksi, the tensile strength equal to 350 psi, the yield strength (f_y) of steel 40 ksi and its modulus of elasticity (E_s) equal to 29000 ksi.

The M-ϕ relationship for the T-beam (Fig. 4) shows an almost linearly elastic initial portion. The stiffness is reduced when the concrete cracks due to tension, but a more significant change occurs due to the yielding of

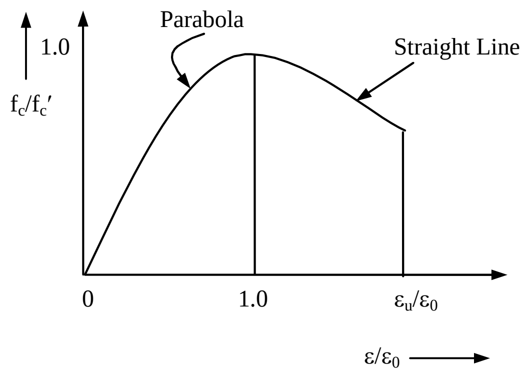

Fig. 1: Hognestad Model for Concrete

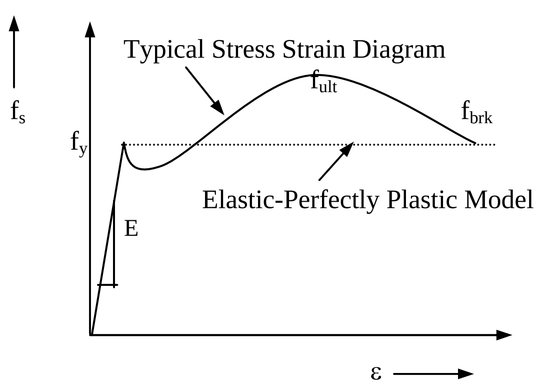

Fig. 2: Stress-Strain Diagram for Steel

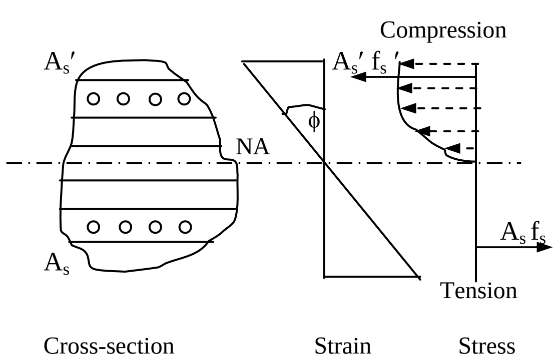

Fig. 3: Derivation of M-ϕ Relationship for RC Section

reinforcing bars at curvature of ± 0.00216 radian/ft, corresponding to bending moment +60 k-ft or –75 k-ft. Here, the negative moment capacity is larger because of the greater amount of negative (top) steel. However due to the ductility of steel, the section does not fail until the concrete itself begins to lose strength at a strain of 0.002 or crushes at 0.003. A similar relationship for the rectangular column section (Fig. 5) also shows little 'kinks' when the concrete cracks due to tension, but the yielding of the reinforcing bars occurs at curvature ±0.00324 radian/ft, corresponding to bending moment ±28 k-ft. Once again, the section does not fail up to curvatures of ±0.015 radian/ft, as shown in Fig. 5.

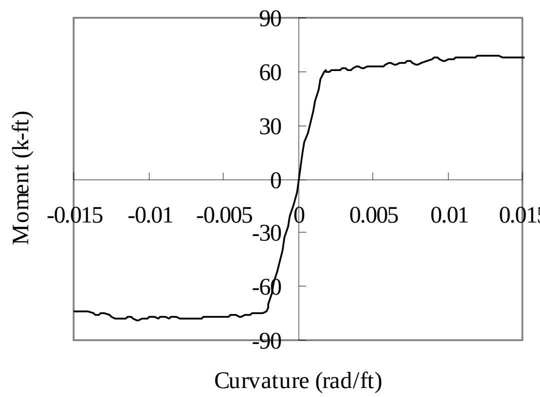

Fig. 4: Moment vs. Curvature for T-Beam

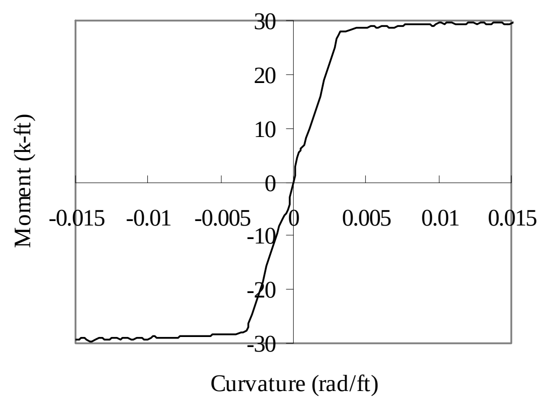

Fig. 5: Moment vs. Curvature for Column

However structural members are often subjected to significant axial forces. In fact for typical RCC structures, columns are sometimes designed to resist axial forces only. Fig. 6 and 7 show families of M-ϕ curves for the 10″×10″ square column section analyzed earlier. Instead of the earlier analysis based on no axial force, the column is now subjected to compressive forces (P =) 25, 50, 75, 100, 200 and 300 kips. For clarity of the presentation, the results are presented in two different figures. Fig. 6 shows the results for the smaller axial forces (25, 50 and 75 kips), while Fig. 7 shows the results for the larger forces (100, 200 and 300 kips). The results show that the column loses ductility if the axial load is increased, which is due to the increased compressive strain on concrete that pushes is closer to failure. At greater axial loads, the M-ϕ diagram tends to go downwards (indicating failure) at smaller curvatures. For example, this critical curvature is beyond the range of the graph for P = 25 and 50 kips, but at 75 kips, this curvature is clearly shown to be 0.010 radian/ft. At 100, 200 and 300 kips, the critical curvature decreases to 0.0079, 0.0047 and 0.0032 radian/ft respectively. However, the initial stiffness and moment capacity of the section increases for P up to 100 kips in the cases studied here (from 28 to 35, 43, 47 and 52 k-ft), beyond which the ultimate moment capacity decreases (to 44 and 23 k-ft respectively). This is explained by the 'failure' modes of RCC column.

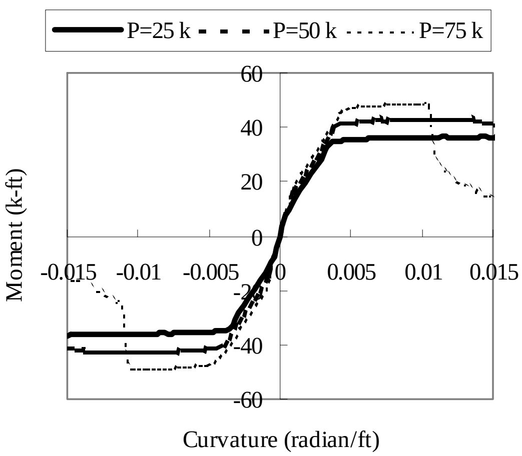

Fig. 6: Moment vs. Curvature for
P = 25~75 k

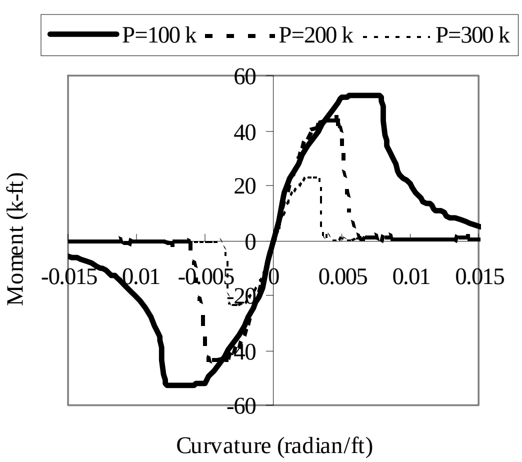

Fig. 7: Moment vs. Curvature for
P = 100~300 k

The design strength of column (or other RCC members) can be governed by two modes of 'failure'; i.e., the yielding of steel or the crushing of concrete. If the applied bending moment is 'small' compared to the axial load, the column fails by the crushing of concrete; while the second mode governs for comparatively smaller loads and larger bending moments. In between, there is a balanced load P_b and corresponding moment M_b when these two 'failures' occur simultaneously. Thus, if the axial load (P) is greater than P_b, the column fails by concrete crushing while the opposite happens if P is smaller than P_b. Also, the column moment capacity increases if P is increased from 0 to P_b while it decreases if P is increased beyond P_b. Details of such P-M 'Interaction diagrams' are available in standard RCC texts and are not repeated here. However, it can be concluded that the effect of axial loads on the M-ϕ relationship for RCC members (particularly columns) is very important.

Nonlinear Structural Responses

Based on the nonlinear structural properties derived in the previous sections, the nonlinear response of a simple 2-dimensional RCC frame is studied. The frame is actually 3-dimensional, with the members in a direction perpendicular to the paper also contributing substantial loads on the columns. The 2-storied structure (Fig. 8.) is made of the same beam and column as were studied earlier and it is subjected to vertical load on the beams. In this first analysis, the M-ϕ relationships for the columns are chosen for the case without axial load. The variation of the vertical deflection at the midspan (B) of the top floor beam for different values of the vertical load (w) is shown in Fig. 9. This figure shows the relative importance of different nonlinearities on the structural response. The deflection is much less (and almost linear elastic) if only the geometric nonlinearities are included in the analysis. However, if only the material nonlinearities are considered, the midspan deflection suddenly increases when the flexural capacity of the T-beam section is exhausted; i.e. at a load of about 5 k/ft. Although loads as high as 5 k/ft or greater are rare in normal RCC frames, these are nevertheless included for the purpose of illustration. At even higher loads (about 7 k/ft), the geometric nonlinearity tends to stiffen the beam somewhat, so that the deflection at B does not increase as much as before. However, in the absence of such stiffening (i.e., if the geometric nonlinearities are neglected), the midspan deflection diverges markedly when the flexural capacity of the beam is reached.

As mentioned, these results are obtained for columns without axial load. If the effect of axial load on the column properties is included, the results can be different. Fig. 10 shows that if the effect of column axial load is neglected in the M-ϕ relationship, the stiffness of the structure can be different, and the analysis may not be able to predict the structural failure due to the exhaustion of column capacity (i.e., a sharp increase in deflection in Fig. 10).

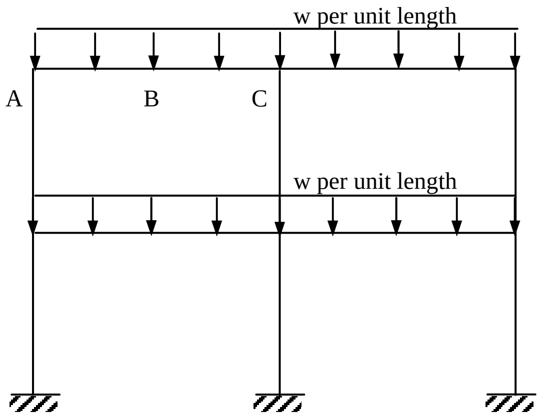

Fig. 8: 2-Storied Building Under Vertical Load

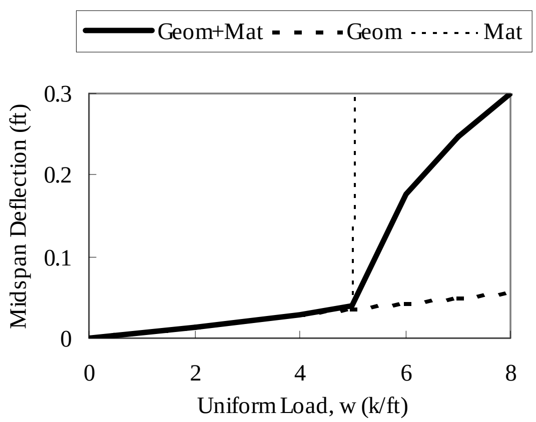

Fig. 9: Midspan Deflection vs. Load

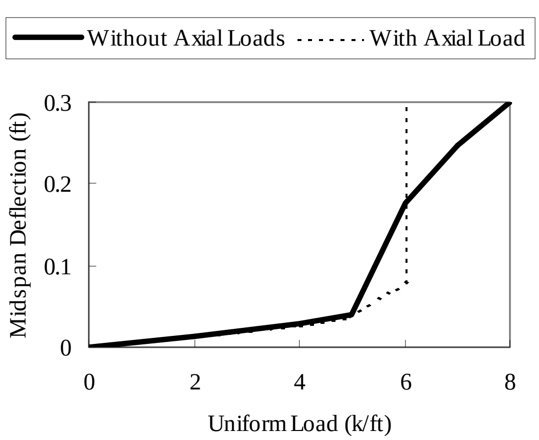

Fig. 10: Vertical Deflection for Different Column Properties

failure pattern of RCC structures rather than the flexural strength. For an RCC member, it is a combination of the shear capacity of the concrete and lateral reinforcement and can be calculated from empirical equations suggested in the American Concrete Institute (ACI) Code. Since it depends on the axial load, neglecting the axial load in determining member properties can lead to erroneous estimation of the shear strength. Fig. 11 shows that although the flexural capacity of the T-beam under study is still not exceeded, it fails in shear when the uniformly distributed vertical load is only about 3.55 k/ft.

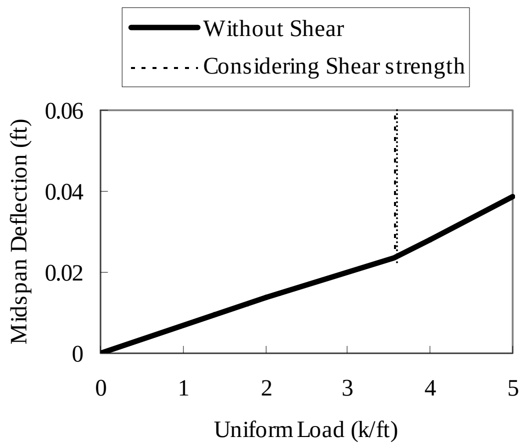

Fig. 11: Effect of Shear Strength

Results from Linear and Nonlinear Dynamic Analyses

The nonlinear properties of RCC can be used along with a suitable time-step numerical integration scheme (Clough & Penzien 1975, Chopra 1998) to evaluate the nonlinear dynamic response of RCC structures. For dynamic response, the elasto-plastic stress-strain relationship can be extended for loading and unloading, so that the stress-strain diagram can be approximated by a hysteretic closed loop curve. The 2-storied model frame is used here again to observe the difference between the linear and nonlinear response to seismic vibrations. In addition to the uniformly distributed vertical load on beams (w = 1.5 k/ft), now the structure is also subjected to El Centro (1940) and Kobe (1995) earthquake vibrations.

Only the shear force in the ground floor central column is considered here for illustration. Fig. 12 and 13 show the results from linear and nonlinear analyses respectively for El Centro earthquake vibration. The ground motion used for study here had maximum ground acceleration of about 0.3g. Other than minor differences, the time-series variations of shear forces are similar in both the linear and nonlinear cases. The maximum shear force in the central column is about 13 kips, which is within the shear capacity of the column. Therefore, the column does not fail in shear.

Also, no visible shift in the mean response is noted (which remains almost zero throughout), indicating no major yielding of the materials.

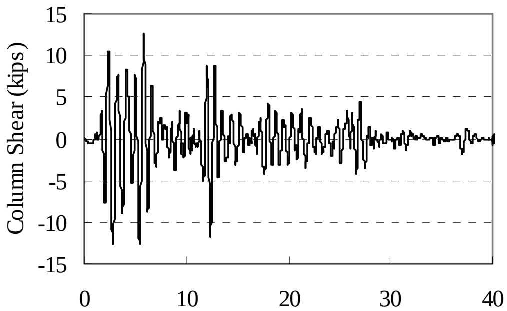

Fig. 12: Linear Response to El Centro Earthquake

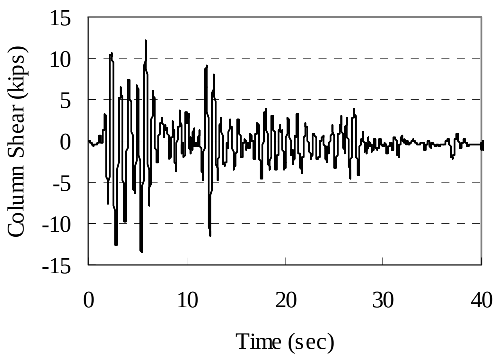

Fig. 13: Nonlinear Response to El Centro Earthquake

The results from Kobe earthquake vibration are shown in Fig. 14 and 15. Here the maximum ground acceleration amplitude is about 0.55g, which causes significant nonlinear response of the structure under study. The linear and nonlinear shear forces in the ground floor column also show significant differences. The maximum shear force for the nonlinear system is about 35 kips, which is more than 30% greater than the linear response. Also, there is a noticeable shift in the mean shear of the column (from zero to about -8 kips) indicating significant yielding of the column. A very important aspect of the nonlinear response is that the maximum shear force is now greater than the shear force capacity of the column (i.e., 19 kips), which means that the column would fail in shear.

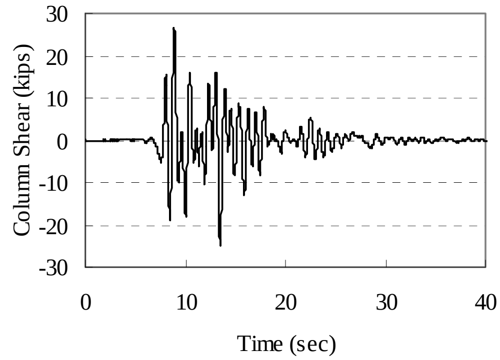

Fig. 14: Linear Response to Kobe Earthquake

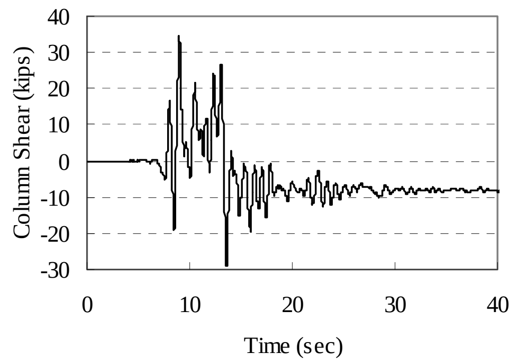

Fig. 15: Nonlinear Response to Kobe Earthquake

Conclusions

The main conclusions of this study are

1. Both the geometric and material nonlinearities can be important in evaluating the structural responses of RCC structures.
2. The Moment-Curvature relationship of RCC members strongly depends on the axial load.
3. The shear strength may govern the failure modes of RCC members in many cases, particularly in seismic vibrations.

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